



EXCITATION CROSS SECTION FOR SOME OF THE  
DOUBLY IONIZED STATES OF ARGON, NEON  
AND KRYPTON BY FAST ELECTRONS

S.H. Koozekanani

The Ohio State University  
**ElectroScience Laboratory**

(formerly Antenna Laboratory)  
Department of Electrical Engineering  
Columbus, Ohio 43212

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## ABSTRACT

Excitation cross section for the doubly ionized states of argon, neon and krypton has been found for the process where a fast moving free electron impinges upon a neutral atom causing the neutral atom to go from  $l^N$  to  $(l^{N-3} - nl'')$  in a very short time compared to the relaxation time of the ion.

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# EXCITATION CROSS SECTION FOR SOME OF THE DOUBLY IONIZED STATES OF ARGON, NEON AND KRYPTON BY FAST ELECTRONS

## I. INTRODUCTION

It has been shown<sup>1,2,3</sup> that the method of sudden perturbation can account for the excitation of some of the excited states of argon II in the pulsed argon laser. This method of calculation yields correct results only when the Hamiltonian changes from an initial state to a final state in a very short time compared to the relaxation time of the system.<sup>5</sup> In this method, the eigenfunctions of the unperturbed Hamiltonian are expanded in terms of the eigenfunctions of the perturbed system. In the problem that we are considering, we assume that a neutral atom with an  $\ell^N$  outer shell configuration loses  $m(m=2)$  of its  $\ell^N$  electrons, in a very short time compared to the relaxation time of the atom, after making a collision with a fast impinging electron. Before collision, the eigenfunction of the system consists of the product of a free electron wave function with that of the product of a free electron wave function with that of the bound atomic electrons. After collision, the new eigenfunctions are the product of the wave function of  $(m+1)$  free electron, with those of  $\ell^{N-m}$  bound atomic electrons. Two cases are considered. For the first case, it is assumed that the atom makes a collision with a fast-moving electron, loses two of its  $N$  outer shell electrons, and is simultaneously excited, i.e.,  $\ell^N \rightarrow (\ell^{N-3-n\ell})$ . There the atom is doubly ionized and excited at the same time. For the second case, it is assumed that two collisions are involved, i.e., the atom makes a collision and goes to an excited single ionized configuration  $(\ell^{N-2-n\ell'})$ ; then, though another collision, it may lose its excited  $n\ell'$  electron or one of the  $\ell^{N-2}$  core electrons. This process is important when  $(\ell^{N-2-n\ell'})$  is a metastable state or a state with relatively long life time. The states for which excitation cross sections will be found are  $|\ell^3, \phi_2, n\ell_s''; {}^3P_J\rangle$ ,  $|\ell^3, \phi_2, n\ell_s''; {}^1D_2\rangle$  and  $|\ell^3, \phi_2, n\ell_s; {}^1S_0\rangle$  with  $\phi_2 = {}^4S, {}^2D$  or  ${}^2P$ .

To obtain the probability of production of any one of the new states, we have to expand the unperturbed Hamiltonian eigenfunctions in terms of the complete set of eigenfunctions of the new Hamiltonian. The overlap integrals in these calculations involve those between electronic bound states before and after collision and between bound and resulting free electronic states. The absolute-square values of the former set of integrals are proportional to the excitation cross section, whereas the corresponding values for the latter set are proportional to the ionization cross section. Since experimental double ionization cross sections for

most of the atoms are known, the excitation cross section for case one can be calculated exactly.

For example, for the  $3p^6$  outershell configuration of neutral argon to go to the  $3p^3 - 4p$  state after collision with a 100 ev electron, we find a cross section of  $8.9 \times 10^{-19} \text{ cm}^2$ . In what follows we shall attempt to find expressions for the cross sections of the various excited states. These states are represented by their angular momentum quantum numbers, i.e., states of the form  $|\ell^{N-3}\phi_2; n\ell''s; \phi_1\rangle$ . For noble gas atoms where  $N=6$ ,  $\ell=\ell''=1$  we have  $\phi_2 \equiv {}^4S, {}^2P$  or  ${}^2D$  and  $\phi_1$  can be  ${}^1S_1, {}^3P_{0,1,2}$  or  ${}^1D_2$ . We shall calculate cross sections for all the 12 possible combinations of  $\phi_2$  and  $\phi_1$ . Figure 1.

## II. CALCULATION

### Case One

We now assume that the atom makes a collision with an electron and loses two of its electrons, simultaneously. In order to know the possible excited states, we have to expand the unperturbed eigenfunctions  $|\ell^{N-3}\phi_0; F_{1i}\rangle$  in terms of the eigenfunctions of the perturbed Hamiltonian. This eigenfunction consists of the product of  $|\ell^{N-3}\phi_0\rangle$  bound wave function and the  $F_{1i}$  free electron wave function. We have, after expansion

$$(1a) \quad |\ell^{N-3}\phi_0; F_{1i}\rangle = \sum_{n,f} a_{nf} |\ell_n^{N-3}\phi_2; n\ell''s, \phi_1'; F_{1f}, F_{2f}, F_{3f}\rangle ,$$

where  $|\ell_n^{N-3}\phi_2; n\ell''s, \phi_1'\rangle$  is an excited doubly ionized state of the atom,  $n\ell''$  is the excited electron, and  $\ell_n$  are the core electrons. The subscript  $n$  indicates that the radial wave function of  $\ell_n^{N-3}$  core electrons changes as the principal quantum number  $n$  of the excited electron assumes different values. In Eq. (1a),  $s$  stands for the spin  $1/2$  of the electron, and  $\phi$  with or without super- or subscript stands for the corresponding angular wave functions. For example, in  $\ell^{N-3}\phi_2$ ,  $\phi_2$  replaces  $L_2, S_2, M_{L_2}$  and  $NS_2$ , where  $L_2$  is the total orbital angular momentum,  $S_2$  is the total spin-angular momentum, and  $M_{L_2}$  and  $NS_2$  are the projections of  $L$  and  $S$  on the magnetic  $z$ -axis.  $F_{1f}, F_{2f}$  and  $F_{3f}$  represent the wave functions for the three free electrons after the impact. The summation in Eq. (1a) is over all the possible final states,  $F$ . For Eq. (1) we have

$$(1b) \quad \sum_f a_{nf} = \sum_f \langle \ell_n^{N-3} \phi_2, n\ell''s, \phi_1' : F_{1f}, F_{2f}, F_{3f} | \ell^{N\phi_0} : F_{1f} \rangle .$$

To calculate these inetrrix elements, we first expand the bound atomic wave function  $|\ell^{N\phi_0}\rangle$  in terms of its parents and grandparents and then decouple two electrons, in other words we have

$$(2a) \quad |\ell^{N\phi_0}\rangle = \sum_{\phi, \phi_1} (\ell^{N-1}\phi \{|\ell^{N\phi_0}\rangle) (\ell^{N-2}\phi_1 \{|\ell^{N-1}\phi\} \\ |\ell^{N-2}\phi; \ell s; \ell s, \phi_0\rangle$$

where two electrons have been separated from the core. The eigenfunctions in Eq. (2a) can be decoupled to give us,

$$(2b) \quad |\ell^{N\phi_0}\rangle = \sum_{\substack{\phi, \phi_1, M_L, M_{L_1}, \\ N_S, N_S, N_{S_1}, M_{\ell_2}, N_{S_2}}} (-)^{L+L_1-2\ell+M_{L_0}+M_L+S+S_1-2S+N_{S_0}+N_S}$$

$$\{[L_0] [L] [S_0] [S]\}^{\frac{1}{2}} (\ell^{N-1}\phi \{|\ell^{N\phi_0}\rangle) (\ell^{N-2}\phi_1 \{|\ell^{N-1}\phi\}$$

$$\begin{pmatrix} L & \ell & L_0 \\ M_L & M_{\ell_1} & M_{L_0} \end{pmatrix} \begin{pmatrix} L_1 & \ell & L \\ M_{L_1} & M_{\ell_2} & M_L \end{pmatrix} \begin{pmatrix} S & s & S_0 \\ N_S & N_{S_1} & N_{S_0} \end{pmatrix} \begin{pmatrix} S_1 & s & S \\ N_{S_1} & N_{S_2} & N_S \end{pmatrix}$$

$$|\ell^{N-2}\phi_1\rangle = |\ell s, M_{\ell_1} N_{S_1}\rangle = |\ell s, M_{\ell_2} N_{S_2}\rangle ,$$

where the symbol  $[x] = 2x+1$ ,  $\phi_n$  stands for  $L_n, S_n, M_{L_n}$  and  $N_{S_n}$ . Replacing Eq. (2a) back in Eq. (1) we obtain

$$\begin{aligned}
(2c) \quad \sum_f a_{nf} = & \sum_{\phi, \phi_1} (\ell^{N-2}\phi_1 \{ \ell^{N-1}\phi \} (\ell^{N-1}\phi \{ \ell^N\phi_0 \} \\
& \langle \ell^{N-3}_{n\phi_2, n\ell''s, \phi_1'} | \ell^{N-2}\phi_1 \rangle \\
& \times \left\{ \sum_{\substack{M_L, M_{\ell_1}, M_{L_1}, M_{\ell_2} \\ N_S, N_s, N_{S_1}, N_{S_1}, f}} (-1)^{L+L_1-2\ell+M_{L_0}+M_L+S+S_1-2s+N_{S_0}+N_S} \right. \\
& \{ [L_0] [L] [S_0] [S] \}^{\frac{1}{2}} \\
& \left( \begin{matrix} L & \ell & L_0 \\ M_L & M_{\ell_1} & M_{L_0} \end{matrix} \right) \left( \begin{matrix} L_1 & \ell & L \\ M_{L_1} & M_{\ell_2} & M_L \end{matrix} \right) \left( \begin{matrix} S & s & S_0 \\ M_S & M_{S_1} & M_{S_0} \end{matrix} \right) \left( \begin{matrix} S_1 & s & S \\ M_{S_1} & M_{S_2} & M_S \end{matrix} \right) \\
& \times \left. \left. \left. \langle f_{1i} | f_{1f} \rangle \langle \ell_s, M_{\ell_1}, N_{S_1} | f_{2f} \rangle \langle \ell_s, M_{L_2}, N_{S_2} | F_{3f} \rangle \right\} \right.
\end{aligned}$$

In the above equation the absolute square value of the quantity in braces ( $\{\}$ ) is proportional to the double ionization probability of the atom. It consists of the product of an initially free electron and two bound electrons with the wave functions of three free electrons. In Eq. (2c), one more matrix element remains to be calculated, and that is the quantity

$$\begin{aligned}
(2d) \quad & \langle \ell^{N-3}_{n\phi_2; n\ell'', \phi_1'} | \ell^{N-2}\phi_1 \rangle \\
& = \sum_{\substack{\phi_2', M_{L_2}, M_{\ell''}, M_{L_2'} \\ M_{\ell} N_{S_2}, N_{S_2'}, N_S}} (-1)^{L_2+L_2'-\ell''-\ell+M_{L_1'}+M_{L_1}+S_2+S_2'-2s+M_{S_1'}+M_{S_1}} \\
& \{ [L_1'] [L_1] [S_1'] [S_1] \}^{\frac{1}{2}} \times
\end{aligned}$$



$$\begin{aligned}
& \times \begin{pmatrix} L_2 & \ell'' & L_1' \\ M_{L_2} & M_{\ell''} & M_{L_1'} \end{pmatrix} \begin{pmatrix} S_2' & s & S_1' \\ N_{S_2} & N_s & N_{S_1'} \end{pmatrix} \begin{pmatrix} L_2' & \ell & L_1 \\ M_{L_2'} & M_{\ell} & M_{L_1} \end{pmatrix} \begin{pmatrix} S_2' & s & S_1 \\ N_{S_2'} & N_s & N_{S_1} \end{pmatrix} \\
& \times (\ell^{N-3}\phi_2 \{ \ell''^{N-2}\phi_1 \} \langle n \ell'' s, M_{\ell''} N_s | \ell s, M_{\ell} N_s \rangle \\
& \langle \ell_n^{N-3}\phi_2 | \ell^{N-3}\phi_2 \rangle
\end{aligned}$$

or

$$\begin{aligned}
(2e) \quad & \langle \ell_n^{N-3}\phi_2, n \ell'' s, \phi_1' | \ell^{N-3}\phi_1 \rangle = (\ell^{N-3}\phi_2 \{ \ell^{N-2}\phi_1 \} \int_0^\infty f_{\ell}(r) F_{n \ell''}(r) dr \\
& \times \left[ \int_0^\infty f_{\ell}(r) F_{\ell n}(r) dr \right]^{N-3} \delta(\phi_1, \phi_1') \delta(\phi_2, \phi_2') \delta(\ell, \ell'') \delta(\ell_n, \ell) .
\end{aligned}$$

Of course, the results of Eq. (2d) could have been intuitively clear except for the coefficient in front of the integrals. In Eq. (2e)  $f_{\ell}(r)$  is the radial wave function of one of the  $|\ell^{N-3}\phi_0\rangle$  neutral ground state atoms;  $f_{n \ell}(r)$  and  $F_{\ell n}$  are the wave functions of the excited running electron and the corresponding core electrons of the  $|\ell_n^{N-3}\phi_2, n \ell'' s, \phi_1'\rangle$  doubly ionized configuration. In Eqs. (2a) through (2c) the quantities  $(\ell^{N-1}\psi \{ \ell^{N-1}\phi \})$  are the coefficients of fractional parentage (6) and the symbols in large parentheses are the usual 3 - j symbols.<sup>7</sup>

The probability that the doubly ionized atom will be in an excited state of the form  $|\ell_n^{N-3}\phi_2, n \ell'' s, \phi_1'\rangle$  would be proportional to  $|a_n|^2$ , where

$$(3a) \quad a_n = \sum_f a_{nf} .$$

This means we have to calculate the quantities in braces of Eq. (2c) or, in other words, sum over expressions of the form  $\langle f_2 | \ell s M_{\ell} M_s \rangle$ , which are the overlap integrals between the bound electronic states and all the possible final states of the free electron. We can, however, avoid these calculations by using the experimentally available total cross section data for removing two electrons from the atom. If we represent the double ionization cross section by  $Q^{++}(E)$ , where  $E$  represents the energy of the initial free electron state  $|f_{i1}\rangle$ , then from Eqs. (2c), (2e), and (3a)

we have for the excitation cross section of the state  $|\ell N-3\phi_2, n\ell'', \phi_1\rangle$  from the ground state  $|\ell N\phi_0\rangle$ , after impinging with an electron having a kinetic energy  $E$ ,

$$(3b) \quad Q_n(E) = \left\{ \sum_{\phi} (\ell N-3\phi_2 \{ |\ell N-2\phi_1\rangle (\ell N-2\phi_1 \{ |\ell N-1\phi\rangle (\ell N-1\phi \{ |N\phi_0\rangle \right. \\ \times \int_0^{\infty} f_{\ell}(r) f_{n\ell''}(r) dr \times \left[ \int_0^{\infty} f_{\ell}(r) f_{\ell n}(r) dr \right]^{N-3} \left. \right\}^2 \\ Q^{++}(E) \delta(\ell, n\ell'') \delta(\ell, \ell n),$$

where  $Q_n(E)$  gives the excitation cross section for the  $|\ell N-3\phi_2, n\ell'', \phi_1\rangle$  configuration, the  $N-3$   $\ell_n$  electrons couple to give  $\phi_2$  i.e.,  $L_2, S_2$ ; then these angular momenta couple with  $\ell''$  to give  $L_1, S_1, M_1, N_1$  or  $\phi_1$ . For the case of Noble gas atoms, which are of interest to this paper,  $\ell = \ell_n = n\ell'' = 1$ ,  $N = 6$  and the only possible  $L_2 S_2$  states are  $^2D, ^2P$  or  $^4S$ .<sup>8</sup> The possible final  $L_1 S_1$  states would be  $^1D, ^3P$  or  $^1S$ . This is seen from Eq. (2e). Moreover, the summation over  $\phi$  in Eq. (3b) breaks down since in the case of  $\ell^5$  only one possible  $\phi$  state can exist, namely  $^2P$ .

So far neither the coefficients of fractional parentage nor anything in Eqs. (3b) or (3c) gives us any information about the total angular momentum  $J = L + S$  of the excited state or its excitation cross section. In the above example, for the noble gas atoms we find that the only possible excited states, when two electrons are removed from an outer  $p^6$  shell, are  $|\ell^3\phi_2, n\ell'', ^1D\rangle$ ,  $|\ell^3\phi_2, n\ell'', ^3P\rangle$ , or  $|\ell^3\phi_2, n\ell'', ^1S\rangle$ , with  $\phi_2$  assuming any of the three possible states of  $\ell^3$ . In this example,  $^1D_J$  has a  $J = 2$  and  $^1S_J$  has a  $J = 0$ , but in  $^3P_J$ ,  $J$  can assume the value of 1, 2 or 0. In general, to find the probability of excitation for any  $J$  we have to change the order of coupling of the angular momenta of the expression in Eq. (2a) from LS to jj coupling, i.e.,

$$(4a) \quad |\ell N\phi_0\rangle = \sum_{\phi_1, \phi} (\ell N-1\phi \{ |\ell N\phi_0\rangle (\ell N-2\phi_1 \{ |\ell N-1\phi\rangle \\ \times |\ell N-2L_1 S_1; \ell_1 s_1, LS; \ell_2 s_2 L_0 S_0 J_0\rangle,$$

$$(4b) \quad |l^N \phi_0\rangle = \sum_{\phi, \phi_1} (l^{N-1} \phi \{ |l^N \phi_0 \rangle (l^{N-2} \phi_1 \{ |l^N \phi \rangle \sum_{J, J_1, j_1, j_2} \{ [L_0] [S_0] [L] [S] [J] [J_1] [j_1] [j_2] \}^{\frac{1}{2}} \begin{Bmatrix} L & l_2 & L_0 \\ S & s_2 & S_0 \\ J & j_2 & J_0 \end{Bmatrix}$$

$$\begin{Bmatrix} L_1 & l_1 & L \\ S_1 & s_1 & S \\ J_1 & j_1 & J \end{Bmatrix} \times |l^{N-2} L_1 S_1 J_1; l_1 s_1 j_1, LSJ; l_2 s_2 j_2, J_0 M_0\rangle.$$

In the above equations  $j_n = l_n + s_n$ , the quantities in curly brackets are the 9-j symbol, and to distinguish the two separated electrons, for the purpose of summation, a subscript is added. As in Eq. (2b) we can uncouple the eigenfunction  $|l^{N-2} L_1 S_1 J_1; l_1 s_1 j_1, LSJ; l_2 s_2 j_2, J_0 M_0\rangle$  in terms of 3-j symbols and a product of the three wave functions, namely,  $|l^{N-2} L_1 S_1 J_1 M_1\rangle |l_1 s_1 j_1 M_{j_1}\rangle |l_2 s_2 j_2 M_{j_2}\rangle$ . Using instead the jj coupled eigenfunctions, Eq. (2c) becomes

$$(4c) \quad \sum_f a_{nfJ_1} = \sum_{\phi, \phi_1, J, j_1, j_2} (l^{N-2} \phi_1 \{ |l^{N-1} \phi \rangle (l^{N-1} \phi \{ |l^N \phi_2 \rangle$$

$$\{ [L_0] [S_0] [L] [S] [J] [J_1] [j_1] [j_2] \}^{\frac{1}{2}} \begin{Bmatrix} L & l_2 & L_0 \\ S & s_2 & S_0 \\ J & j_2 & J_0 \end{Bmatrix} \begin{Bmatrix} L_1 & l_1 & L \\ S_1 & s_1 & S \\ J_1 & j_1 & J \end{Bmatrix}$$

$$\times \langle \ell_n^{N-3} L_2 S_2 j_2 ; n \ell'' s, L_1' S_1' J_1' M_1' | \ell^{N-2} L_1 S_1 J_1 M_1 \rangle \times$$

[ summation over all final states of the overlap integrals and some 3-j symbols] .

In Eq. (4c) we have added an extra index  $J_1$  to the coefficients  $a_{nf}$  to designate the total angular momentum of the excited state. As before, the last term in brackets is simply the overlap integrals and is proportional to the double ionization cross section, and the first portion gives the excitation probability. The excitation cross section to any state of the form  $|\ell_n^{N-3} L_2 S_2 J_2 ; n \ell'' s j, J_1 M_1 \rangle$  from the ground state  $|\ell^{N\phi_0} \rangle$  is,

$$(5) \quad Q_{nJ_1} = \left\{ \sum_{\phi} \sum_{J, j_1, j_2} (\ell^{N-2\phi_2} \{ | \ell^{N-2\phi_1} \rangle (\ell^{N-2\phi_1} \{ | \ell^{N-1\phi} \rangle \right. \\ \left. (\ell^{N-1\phi} \{ | \ell^{N\phi_0} \rangle \right. \\ \times \{ [L_0] [S_0] [L] [S] [J] [J_1] [j_1] [j_2] \}^{\frac{1}{2}} \\ \left. \left\{ \begin{matrix} L & \ell_2 & L_0 \\ S & s_2 & S_0 \\ J & j_2 & J_0 \end{matrix} \right\} \left\{ \begin{matrix} L_1 & \ell_1 & L \\ S_1 & s_1 & S \\ J_1 & j_1 & J \end{matrix} \right\} \int_0^{\infty} f_{\ell}(l) f_{n\ell''}(r) dr \right. \\ \left. \times \left[ \int_0^{\infty} f_{\ell}(r) f_{n\ell}(r) dr \right]^{N-3} \right\}^2 Q^{++}(E) \phi(\ell, \ell_n) \phi(\ell, n\ell'') .$$

For the calculations of the radial integrals, a self-consistent Hartree-Fock computer program in the Slater approximation was used,<sup>10</sup> and the results for neon, argon, and krypton are given in Table I. As an illustration, we have calculated the excitation cross section for all the  $3p^3-4p$  configuration of argon III, and this is presented in Table II.

The total cross section for having a  $3p^3-4p$  configuration after collision with a 100 ev electron is about  $0.88 \times 10^{-18}$ , and this is divided among all the possible angular momentum states permissible from the  $3p^3-4p$  configuration.

## Case Two

Here we assume that the atom loses one electron through collision, and assumes an excited configuration of the form  $(\ell_{1n}^{N-2}, n\ell_1)$ . Then, through another collision, loses the excited  $n\ell_1$  electron or one of the  $\ell^{N-2}$  core electrons to assume a final  $(\ell^{N-3}, n\ell'')$  configuration, where  $N$  is the total number of electrons in the valence shell. Having seen the principle of the calculations for case one, we can now write the cross section without going through the detailed mathematics, the only difference being that one has to go twice through equations of the form Eq. (2c). Figure 2 shows schematically what happens.

In the second collision where the excited running electron is lost, we have for the excitation cross section of the state  $|\ell_{2m}^{N-3} \phi_2, \phi_1\rangle$

$$\begin{aligned}
 (6) \quad Q_1(E, E') = & \sum_{\phi, \phi_1} (\ell^{N-3}_{12} \{ |\ell^{N-2}_{\phi_1} \rangle (\ell^{N-2}_{\phi_1} \{ |\ell^{N-1}_{\phi} \rangle \\
 & (\ell^{N-1}_{\phi} \{ \ell^{N-1}_{\phi_0} \rangle \int_0^\infty f_\ell(r) f_{n\ell_1}(r) dr \\
 & \times \left[ \int f_\ell(r') f_{\ell_{1n}}(r') dr' \right]^{N-2} \\
 & \times \int_0^\infty f_{\ell_{1n}}(r'') f_{m\ell_2}(r'') dr \times \left[ \int_0^\infty f_{\ell_{1n}}(r) f_{\ell_{2m}}(r) dr \right]^{N-3} \Bigg\}^2 \\
 & Q^+(E) Q_{n\ell_1}^+(E') \delta(\ell, \ell_{1n}) \delta(\ell, n\ell) \delta(\ell, m\ell_2) \delta(\ell, \ell_{2m}) ,
 \end{aligned}$$

where  $Q^+(E)$  is the usual single ionization cross section and  $Q_{n\ell_1}^+(E')$  is the ionization cross section for removing the  $n\ell_1$  excited electron from the  $(\ell_{1n}^{N-2}, n\ell_1)$  configuration.  $Q_{n\ell_1}$  could be very large as the excited  $n\ell_1$  electron assumes higher orbits. The first set of the two integrals in Eq. (6) gives the overlap integrals for the bound states when after a collision  $\ell^N \rightarrow (\ell_{1n}^{N-2}, n\ell_1)$ , while the second set of the integrals give the overlap integrals for the  $(\ell_{1n}^{N-2}, n\ell_1) \rightarrow (\ell_{1n}^{N-2}) \rightarrow (\ell_{2m}^{N-3}, m\ell_2)$  situation. The possible final states are exactly the same as those given in case one, namely,  $|\ell^3 \phi_2; m\ell_2 s, \phi_1\rangle$ , where  $\phi_1$  is a  $^3P$ ,  $^1D$ , or a  $^1S$  state. Tables III and IV give the radial integrals for the above two processes.

Now, if in the second collision, instead of the excited electron a core electron is removed, i.e., if  $(\ell_{in}^{N-2}, n\ell_1)$  goes into the intermediate state  $(\ell_{in}^{N-3}, n\ell)$ , the total orbital and spin angular momenta of the final states will not be evident as in the previous cases. We have to change the order of coupling of the electrons of the excited singly ionized atom. After having separated the electron which has to be lost from the core i.e., by expressing the state in terms of coefficients of fractional parentage,

$$(7a) \quad |\ell_{in}^{N-2} L_1 S_1; n\ell_1 s, LSJ\rangle = \sum_{\phi_2} (P^{N-3}_{\phi_2} \{ |P^{N-2}_{\phi_1} \rangle \\ \times |\ell_{in}^{N-3} L_2 S_2; \ell_{1n}s, L_1 S_1; n\ell_1 s, LSJ\rangle ,$$

we then change the order by first coupling the excited  $n\ell_1 s$  electron to the  $L_2 S_2$  core states and then couple the separated  $\ell_{1n}s$  core electron to the resultant states to obtain the final LSJ states, i.e.,

$$(7b) \quad |\ell_{in}^{N-2} L_1 S_1; n\ell_1 s, LSJ\rangle = \sum_{\phi_2} (P^{N-3}_{\phi_2} \{ |P^{N-2}_{\phi_1} \rangle \sum_{\bar{L} \bar{S}} \{ [\bar{L}] [\bar{S}] \}^{\frac{1}{2}} \\ \left\{ \begin{matrix} L_2 & L & \bar{L} \\ n\ell_1 & \ell_{1n} & L_1 \end{matrix} \right\} \left\{ \begin{matrix} S_2 & S & \bar{S} \\ s & s & S_1 \end{matrix} \right\} |\ell_{in}^{N-3} L_2 S_2; n\ell_1 s, \bar{L} \bar{S}; \ell_{1n}s, LSJ\rangle$$

where the quantities in large curly brackets are the 6-j symbols.<sup>8</sup> Now we use Eq. (7b) to calculate the overlap integrals between this singly ionized state and the doubly ionized states. The matrix element involved is,

$$(7c) \quad \langle \ell_{in}^{N-2} \phi_1; n\ell_1 s, \phi; F_{1i} | \ell_{2m}^{N-3} \phi_2; m\ell_2 \phi_1'; F_{1f}, F_{2f} \rangle \\ = \left\{ \alpha \sum_f \langle \ell_{1n}s | F_{2f} \rangle \langle F_{1i} | F_{1f} \rangle \right\} \\ \times \int_0^\infty f_{n\ell_1}(r) f_{m\ell_2}(r) dr \times \left[ \int_0^\infty f_{\ell_{1n}}(r) f_{\ell_{2m}}(r) \right]^{N-3} \\ \delta(\phi_1, \bar{\phi}) \delta(n\ell_1, m\ell_2) \delta(\ell_{1n}, \ell_{2m})$$

where  $\alpha$  replaces all the 3-j symbols and summations over them. It is seen that the only possible excited states are the  $\bar{\phi}$  states, namely  $\overline{LS}$ . From Eqs. (7b) and (7c) we can now write the excitation cross section for all the states  $|\ell_{3m}^{N-3} \phi_2, m\ell_3 s \bar{\phi}\rangle$ , where  $\bar{\phi}$  stands for  $\overline{LS}$ ,

$$\begin{aligned}
 (8) \quad Q_2(E, E) = & \left\{ \sum_{\phi} (\ell^{N-3} \phi_2 \{ |\ell^{N-2} \phi_1 \rangle (\ell^{N-2} \phi_1 \{ |\ell^{N-1} \phi \rangle (\ell^{N-1} \phi \{ |\ell^N \phi_0 \rangle \right. \\
 & [\overline{L}]^{\frac{1}{2}} [\overline{S}]^{\frac{1}{2}} \begin{Bmatrix} L_2 & L & \overline{L} \\ n\ell_1 & \ell_{1n} & L_1 \end{Bmatrix} \begin{Bmatrix} S_2 & S & \overline{S} \\ s & s & S_1 \end{Bmatrix} \\
 & \times \int_0^\infty f_{\ell}(r) f_{n\ell_1}(r) dr \times \left[ \int_0^\infty f_{\ell}(r') f_{\ell_{1n}}(r') dr \right]^{N-2} \\
 & \times \int_0^\infty f_{n\ell_1}(r'') f_{m\ell_3}(r'') dr'' \times \left[ \int_0^\infty f_{\ell_{1n}}(r) f_{\ell_{3m}}(r) dr \right]^{N-3} \Big\}^2 \\
 & Q^+(E) Q_{\ell_{1n}}^+(E') .
 \end{aligned}$$

There  $Q_{\ell_{1n}}^+(E)$  is the cross section for removing an electron from the core of the singly ionized excited state of the  $(\ell_{1n}^{N-2}, n\ell_1)$  configuration. This cross section is proportional to the square-absolute value of the overlap integrals in Eq. (7c) between bound and the free electrons. Table V gives the overlap integrals for the bound atomic states of Eq. (7c) for neon, argon, and krypton and Tables VI and VII gives the necessary coefficients of fractional parentage for the  $p^N$  shells of the Noble gas atoms.

For the case of Noble gas atoms, we can also determine the possible  $\bar{\phi}$  states in Eq. (7a)  $L=1$ ,  $S=1/2$ ,  $\phi_2$  is either  $^4S$ ,  $^2P$  or  $^2D$ . From the triangular relations of the 6-j symbols  $\bar{\phi}$  can be  $^3P$  when  $\phi_2 = ^4S$ ,  $^1, ^3[S, P, D]$  when  $\phi_2 = ^2P$  and finally  $\bar{\phi} = ^1, ^3[P, D]$ , when  $\phi_2 = ^2D$ .

### III. CONCLUSIONS

Two cases of excitation have been discussed. For case one, since the double ionization cross section is large<sup>11</sup>, i.e., of the order of  $10^{-17}$  cm<sup>2</sup> and because of the relatively large overlap integrals, we notice that removing two electrons from the valence shell causes an electron to go to an excited  $n\ell''$  shell with a relatively high probability. From Table I we see that this probability decreases as  $n$  increases.

The probability of the excited electron to go to continuum, that is to say the atom to be triply ionized, after the removal of two electrons, would be  $(1 - \Sigma \text{ probability to go to } n\ell'' \text{ excited shell})$ . For the case of neon, we have  ${}^n Q^{+++}(E) = 0.17 Q^{+++}(E)$ , for argon  $Q^{+++}(E) = 0.045 Q^{+++}(E)$  and for krypton  $Q^{+++}(E) = 0.04 Q^{+++}(E)$ .  $Q^{+++}$  is the cross section for triply ionizing the atom. It is seen that in the measurements of ionization cross section of removing two electrons could possibly cause the atom to be rather triply ionized.

For case two, we treated two special situations. Both of these situations are not likely to occur in discharges of Noble gas atoms since the lifetime of the lower excited states are relatively short; however, in atoms where the excited states have longer lifetimes this situation could become probable. An example is the  $3p^4[3p] - 3d^2 F$  states of argon. In this case, the atom becomes ionized and excited and then relaxes to the above ionized metastable state. If the population of these states is large, then it is quite possible that the atom becomes doubly ionized through these states. Labuda, et. al.<sup>12</sup>, have measured the population density of these states and found them to be quite large.



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TABLE I  
OVERLAP INTEGRALS FOR BOUND STATES FOR THE PROCESS  $1N \rightarrow l_n^{N-3}$ ,  
 $nl_1$  FOR NOBLE GAS ATOMS  $N = 6$  AND  $n$  IS THE PRINCIPLE QUANTUM  
NUMBER OF THE EXCITED ELECTRON  $l$ ,  $ln$  and  $nl_1$  ARE ALL p-ELECTRONS

$n$	2	3	4	5	6	7	8
Neon	0.8888	-0.1904	-0.0553	-0.0266	-0.0166		
Argon		0.9612	-0.1723	-0.0460	-0.0222	-0.0133	
Krypton			0.9641	-0.1671	-0.0444	-0.0213	-0.0126

TABLE II  
EXCITATION CROSS SECTION FOR  $|p^3\phi_2, 4p\phi_1\rangle$   
STATES OF ARGON<sup>++</sup> FOR CASE ONE AT  
ELECTRON ENERGIES OF ABOUT 100 EV.

$$Q^{++}(100 \text{ ev}) = 2.96 \times 10^{-17} \text{ cm}^2 \text{ (ref 10)}$$

$Q( (3p^3)^4S, 4p, ^1D_2\rangle) = 2.17 \times 10^{-19} \text{ cm}^2$
$Q( (3p^3)^4S, 4p, ^3P_2\rangle) = 0.95 \times 10^{-19} \text{ cm}^2$
$Q( (3p^3)^4S, 4p, ^3P_1\rangle) = 0.57 \times 10^{-19} \text{ cm}^2$
$Q( (3p^3)^4S, 4p, ^3P_0\rangle) = 0.19 \times 10^{-19} \text{ cm}^2$
$Q( (3p^3)^2D, 4p, ^3P_2\rangle) = 1.20 \times 10^{-19} \text{ cm}^2$
$Q( (3p^3)^2D, 4p, ^3P_1\rangle) = 0.72 \times 10^{-19} \text{ cm}^2$
$Q( (3p^3)^2D, 4p, ^3P_0\rangle) = 0.25 \times 10^{-19} \text{ cm}^2$
$Q( (3p^3)^2P, 4p, ^1D_2\rangle) = 0.72 \times 10^{-19} \text{ cm}^2$
$Q( (3p^3)^2P, 4p, ^3P_2\rangle) = 0.72 \times 10^{-19} \text{ cm}^2$
$Q( (3p^3)^2P, 4p, ^3P_1\rangle) = 0.42 \times 10^{-19} \text{ cm}^2$
$Q( (3p^3)^2P, 4p, ^3P_0\rangle) = 0.13 \times 10^{-19} \text{ cm}^2$
$Q( (3p^3)^2P, 4p, ^1S_0\rangle) = 0.57 \times 10^{-19} \text{ cm}^2$

TABLE III

OVERLAP INTEGRALS FOR THE PROCESS  $l^N \rightarrow l_{1n}^{N-2}, nl_1$ 

$$\left[ \int_0^\infty f_l(r) f_{ln}(r) dr \right]^4 \times \int_0^\infty f_l(r) f_{nl_1}(r) dr$$

	$a_2$	$a_3$	$a_4$	$a_5$	$a_6$	$a_7$
Neon	0.980	-0.1308	-0.0525	-0.0290		
Argon		0.985	-0.1195	-0.0466	-0.0247	
Krypton			0.985	-0.1165	-0.04515	-0.0237

TABLE IV  
OVERLAP INTEGRAL FOR THE CASE  
 $(l_{1n}^{N-2}, nl_1) \rightarrow l_{1n}^{N-2} \rightarrow l_{2m}^{N-3}, nl_2$

$$\left[ \int_0^\infty f_{l_{2m}}(r) f_{l_{1n}}(r) dr \right]^3 \times \int_0^\infty f_{l_{1n}}(r) f_{ml_2}(r) dr$$

Ne II	NeIII				
	$2p^3-2p$	$2p^3-3p$	$2p^3-4p$	$2p^3-5p$	$2p^3-6p$
$2p^4-2p$	0.9894	-0.1177	-0.0452	-0.0247	-0.0153
$2p^4-3p$	0.9996	-0.0476	-0.0227	-0.0133	-0.0088
$2p^4-4p$	0.9992	-0.0416	-0.0213	-0.0128	-0.0086
$2p^4-5p$	0.9992	-0.0397	-0.0209	-0.0127	-0.0085
$2p^4-6p$	0.9992	-0.0390	-0.0207	-0.0126	-0.0085

ArII	ArIII				
	$3p^3-3p$	$3p^3-4p$	$3p^3-5p$	$3p^3-6p$	$3p^3-7p$
$3p^4-3p$	0.9912	-0.1072	-0.0398	-0.0208	-0.0130
$3p^4-4p$	0.9996	-0.0435	-0.0205	-0.0116	-0.0075
$3p^4-5p$	0.9992	-0.0350	-0.0190	-0.0113	-0.0074
$3p^4-6p$	0.9992	-0.0323	-0.0184	-0.0111	-0.0074
$3p^4-7p$	0.9988	-0.0311	-0.0182	-0.0111	-0.0074

KrII	KrIII				
	$4p^3-4p$	$4p^3-5p$	$4p^3-6p$	$4p^3-7p$	$4p^3-8p$
$4p^4-4p$	0.9980	-0.1035	-0.0384	-0.0201	-0.0123
$4p^4-5p$	0.9996	-0.0417	-0.0198	-0.0112	-0.0071
$4p^4-6p$	0.9992	-0.0329	-0.0182	-0.0108	-0.0071
$4p^4-7p$	0.9988	-0.0297	-0.0176	-0.0107	-0.0071
$4p^4-8p$	0.9988	-0.0284	-0.0173	-0.0107	-0.0071

TABLE V  
OVERLAP INTEGRALS FOR THE CASE  
 $(\ell_{1n}^{N-2}, n\ell_1) \rightarrow (\ell_{1n}^{N-3}, n\ell_1) \rightarrow (\ell_{3m}^{N-3}, m\ell_3)$

$$\int_0^{\infty} f_{\ell_{1n}}(r) f_{\ell_{3m}}(r) dr \times \int_0^{\infty} f_{n\ell_1}(r) f_{m\ell_3}(r) dr$$

NeIII						
NeII	2p <sup>3</sup> -2p	2p <sup>3</sup> -3p	2p <sup>3</sup> -4p	2p <sup>3</sup> -5p	2p <sup>3</sup> -6p	
2p <sup>4</sup> -2p	0.9894	-0.1177	-0.0452	-0.0247	-0.0153	
2p <sup>4</sup> -3p	--	0.9341	-0.3680	-0.0842	-0.0477	
-2p <sup>4</sup> -4p	--	--	0.8134	-0.5405	-0.0704	
2p <sup>4</sup> -5p	--	--	--	0.6378	-0.7021	
2p <sup>4</sup> -6p	--	--	--	--	0.4223	

ArIII						
ArII	2p <sup>3</sup> -3p	2p <sup>3</sup> -4p	2p <sup>3</sup> -5p	2p <sup>3</sup> -6p	2p <sup>3</sup> -7p	
3p <sup>4</sup> -3p	0.9912	-0.1072	-0.0398	-0.0208	-0.0130	
3p <sup>4</sup> -4p	--	.9477	- .3402	-0.0768	-0.0438	
3p <sup>4</sup> -5p	--	--	.8444	-0.5102	-0.0653	
3p <sup>4</sup> -6p	--	--	--	0.6828	-0.664	
3p <sup>4</sup> -7p	--	--	--	--	0.4702	

KrIII						
KrII	4p <sup>3</sup> -4p	4p <sup>3</sup> -5p	4p <sup>3</sup> -6p	4p <sup>3</sup> -7p	4p <sup>3</sup> -8p	
4p <sup>4</sup> -4p	0.9920	-0.1035	- .0389	-0.0201	-0.0123	
4p <sup>4</sup> -5p	--	0.9555	-0.3329	-0.0742	-0.0425	
4p <sup>4</sup> -6p	--	--	0.8527	-0.4794	-0.0634	
4p <sup>4</sup> -7p	--	--	--	0.6954	-0.6557	
4p <sup>4</sup> -8p	--	--	--	--	0.4919	

TABLE VI

COEFFICIENTS OF FRACTIONAL PARENTAGE  
FOR STATES  $p^5 \phi$  WITH  $p^4 \phi_1$  PARENTS

$p^5 \phi / p^4 \phi_1$	$1S$	$3P$	$1D$
$2P$	$\sqrt{\frac{1}{15}}$	$\sqrt{\frac{3}{5}}$	$\sqrt{\frac{1}{3}}$

TABLE VII

COEFFICIENTS OF FRACTIONAL PARENTAGE  
FOR STATES  $p^4 \phi_1$  WITH  $p^3 \phi_2$  PARENTS

$p^4 \phi_1 / p^3 \phi_2$	$4S$	$2P$	$2D$
$3P$	$\sqrt{\frac{1}{3}}$	$-\frac{1}{2}$	$-\frac{1}{2}\sqrt{\frac{5}{3}}$
$1D$		$-\frac{1}{2}$	$\sqrt{\frac{3}{2}}$
$1S$		1	



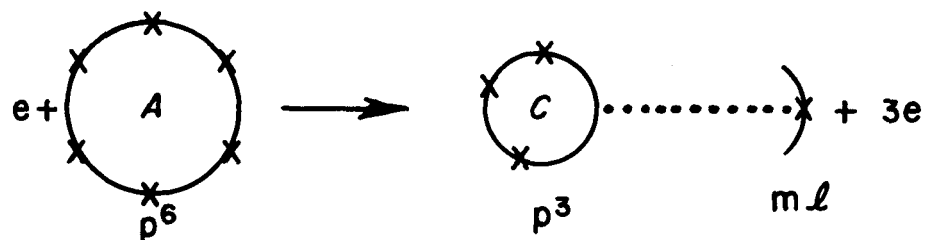


Fig. 1. An electron colliding with a neutral Noble gas atom resulting in three free electrons and an excited doubly ionized atom. For argon if the colliding electron has a kinetic energy of 100 ev the cross section for the  $3p^3-4p$  configuration is  $8.9 \times 10^{-19} \text{ cm}^2$ .

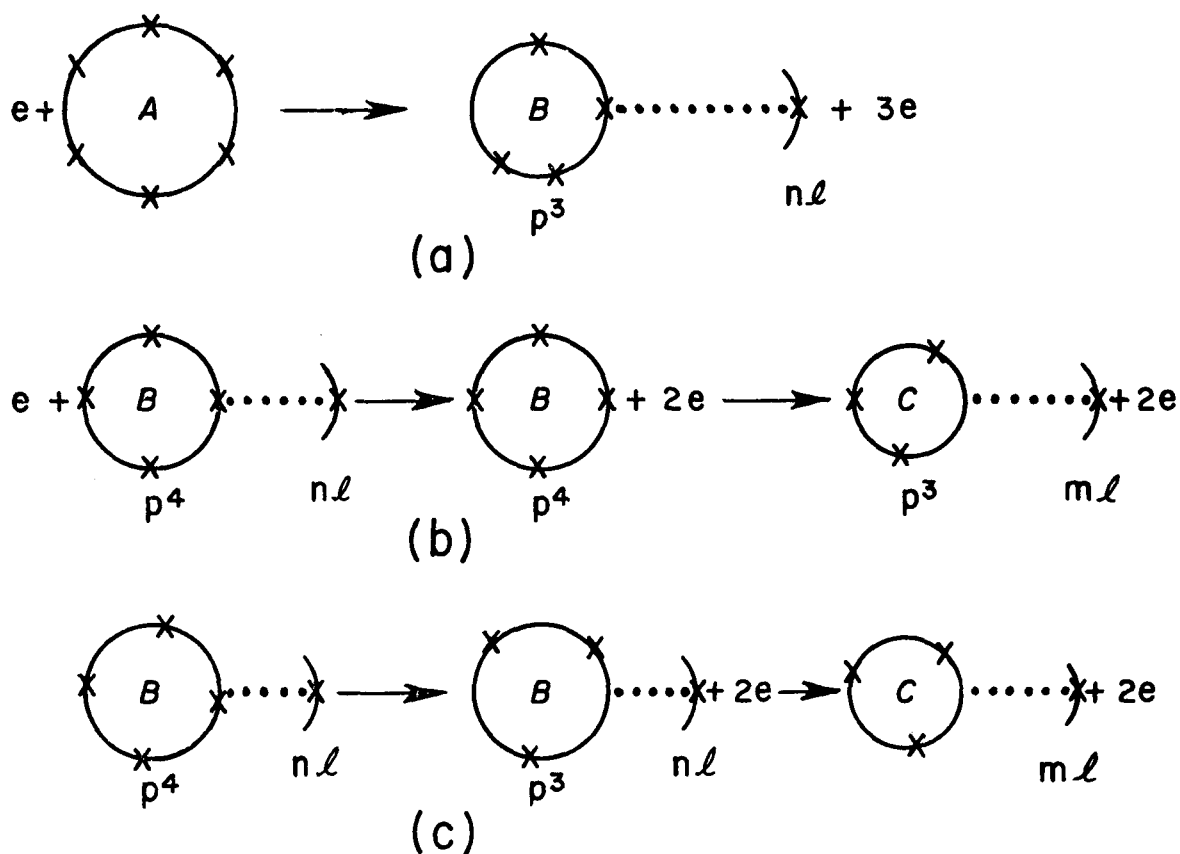


Fig. 2. Here the process of excitation is in two steps.  
 (a) The atom loses an electron and becomes excited to a  $p^4 - nl$  configuration.  
 (b) The excited ionized atom makes another collision, loses its excited electron and then relaxes to a doubly ionized excited state.  
 (c) The excited ionized atom through collision loses one of its core electrons and then relaxes to a doubly ionized excited state.